Research Evaluation NDNS+ 2010–2018

Part A: Self-evaluation

1 Background of the cluster

founding fathers Wiktor Eckhaus, Bert Peletier, and Floris Takens can be seen as the 'founding fathers' of the mathematical field of nonlinear systems in the Netherlands. The formation of the NDNS+ cluster has been a natural step in the evolution of the field of nonlinear systems in the Netherlands.

vision Understanding the emergence of patterns in natural (nonlinear) systems.

mission Understanding the emergence of natural patterns via a combination of nonlinear dynamics, ODE/PDE theory, functional analysis, inverse problems, numerical analysis and scientific computing. Qualitative and quantitative understanding of real-world systems ranging from biological systems, through medical applications to systems in earth science and physics.

strategy The strategy of NDNS+ consists of three parts:

1. Studying the mathematical principles underlying broad classes of dynamical systems, including coupled ODEs, PDEs, (random) maps, delay equations, networks, hybrid systems, etc.
2. Developing computational, multi-scale analysis and dimension reduction techniques for the local and global analysis of large dynamical systems.
3. Combining these fundamental insights and techniques with numerical simulation, observational data and machine learning, to predict, compute and shape the emergence of patterns in real-world systems, both qualitatively and quantitatively.

board The composition of the board is:

Martina Chirilus-Bruckner (UL)	Roeland Merks (CWI)
Jason Frank (UU)	Bob Rink (VUA)
Stephan van Gils, chair, (UT)	Kees Vuik (TUD)
Michiel Hochstenbach (TUE)	Holger Waalkens (RUG)

Within the board there are portfolio managers for: teaching, outreach, acquisition, NDNS+ meeting, and PhD-days.

2 Cluster Research

In this section we describe the current research themes of the cluster. We focus on highlights, and add references to a small selection of other contributions.

2.1 Bifurcation and Chaos

Bifurcation theory and chaos theory have been at the heart of the field of dynamical systems over the last half century. The importance of this subject lies in that it predicts and describes robust dynamical phenomena in a huge number of application areas simultaneously, varying from mechanics, physics and chemistry, to economics and the earth and life sciences.

Bifurcation theory, in particular, aims to classify the scenarios by which phase transitions can take place, and to describe the early warning signals that accompany these transitions. It turns out that many of these scenarios are universal, and that they often entail the onset of complex, e.g. chaotic, behavior.

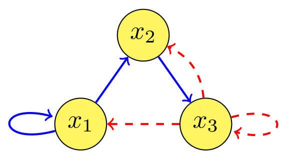
Top publications:[1] [(i),(ii), (iii), (iv)]

Selected other publications: [2, 8, 16, 20, 38, 39, 62]

Highlight: Bifurcations in network dynamical systems

Many systems in science and technology can be viewed as networks: they consist of similar individual nodes with connections between them. Examples include power grids, neuron networks and genetic regulatory networks. It turns out that dynamical systems that exhibit the structure of a network, behave vastly different from general dynamical systems. They may for example synchronize, so that several nodes of the network act in unison (the simultaneous firing of neurons, consensus formation in a decision process.)

A comprehensive mathematical framework for network synchronization was developed by Lee DeVille, Micheal Field, Martin Golubitsky, Eugene Lerman, Ian Stewart et al. throughout the last two decades. The mechanism for this phenomenon was not understood until recently, when Eddie Nijholt, Bob Rink and Jan Sanders (VU Amsterdam) revealed that synchrony breaking bifurcations are governed by so-called *hidden symmetry* [38, 39, 40, 43, 44, 45]. Unlike graph structure, hidden symmetry is compatible with modern dimension reduction techniques (e.g., center manifold reduction), and it also allows for the application of tools from algebra, representation theory and transversality theory.



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$$\frac{dx_2}{dt} = f(x_2, x_1, x_3; \lambda)$$
$$\frac{dx_3}{dt} = f(x_3, x_2, x_3; \lambda)$$

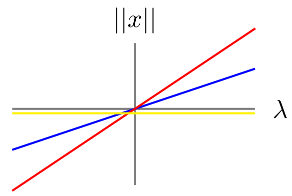

Figure 1: Three steady state branches cross in a synchrony breaking bifurcation for this network. This scenario is impossible in the absence of network structure.

2.2 Multiple Scales

The term *Multiscale Modeling* is used to represent two closely related challenges in the field of nonlinear dynamics of natural systems. Firstly, it refers to an approach where the same physical system is represented at small scale where detail is needed, and at a more coarse-grained system otherwise. Secondly, it can refer to the study of inherently multiscale systems, most particularly biological systems, where aggregate properties can *feed back* on the dynamics of the small scale.

Top publications: [(v),(vi)]

Selected other publications: [53, 31, 54, 9, 65, 42, 15, 14]

[1] Note that there are 'top publications' and 'standard references'.

Highlight: The cross-scale effects of neural interactions during human neocortical seizure activity [17].

Despite enormous efforts to understand the functioning of the brain under normal and pathological conditions, many questions are wide open. Collaboration between the lab of Wim van

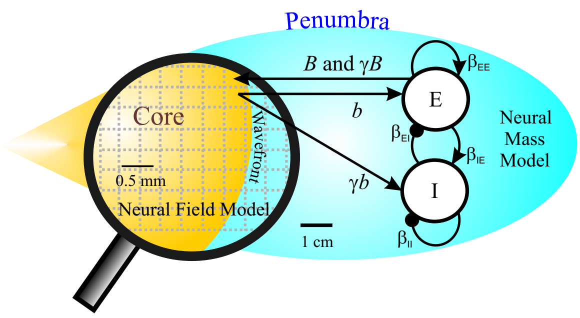

Figure 2: Schematic of the multi-scale model. The model consists of a mesoscopic neural field model for the propagating wavefront that is connected to a neural mass model representing the surrounding macroscopic network. The activation of the inhibitory populations' response, feedforward inhibition, is governed by $\gamma \in [0, 1]$.

Drongelen in Chicago and the Applied Analysis group in Twente, supported by NWO through a visitor grant, has led to a fundamental insight in the role of inhibition in the brain. Brain activity at the millimeter scale registered by a so called *Utah array* was modeled by a neural field, which proved to be a very successful tool since their introduction in the seventies by Hugh Wilson and Jack Cowan. The interaction between the neural field and the neural mass allowed to hypothesize the role of feedforward inhibition in cortex.

2.3 Scientific Computing

Scientific Computing is a fast-growing, highly interdisciplinary field that brings together methods from numerical analysis, high-performance computing and various application fields. It is the area of research that provides better simulation tools aimed at many different applications.

Top publications: [(vii), (viii), (ix), (x),(xi), (xii)]

Selected other publications: [7, 46, 68, 37, 60, 23]

Highlight: High Performance Image Processing.

Within Scientific Computing one of the highlights is the results on image processing, with applications in face recognition, medical imaging, sonar images, seismic images etc. Typical problems are how to improve the accuracy, how to enhance the signal to noise ratio and the development of fast and robust solvers. The standard mathematical tools to solve this type of inverse problems are Bayesian and (regularized) least squares methods. In recent years various breakthroughs have been obtained in the domain of inverse problems. The idea behind this problem is to compare simulations with measurements.

Many imaging problems can be described with the wave equation (ultrasound, MRI, acoustic waves). A popular approach is to consider solutions of the wave equation in the frequency domain. In order to see more details it is important to increase the frequency as much as possible. However for large frequencies, the computing time to find the solution explodes.

In 2006 the Complex Shifted Laplace preconditioner was introduced, which makes it possible to find the solution with only a linear increase in time [19, 61]. Currently, this is the world standard to solve Helmholtz problems. To obtain realistic images the behavior of the solver should be independent of the frequency. To achieve this the group of Kees Vuik collaborates with the group of Reinhard Nabben from the TU Berlin to combine the complex shifted Laplace preconditioner with a second level (deflation) preconditioner [18, 52, 51]. For small to medium size range of frequencies this leads to a solver where the number of iterations is independent of the size of the frequency.

2.4 Variational Methods

In a great many mathematical problems arising from natural systems, the solution can be characterized either as a stationary point of some functional, or as a curve of steepest descent (a gradient flow). The Calculus of Variations, in which functional analysis, geometry, topology and PDE theory interact closely, is the mathematical toolbox *par excellence* for this type of problems, and this field plays a central role in the work of several groups in NDNS+.

Top publications: [(xiii),(xiv), (xv), (xvi),(xvii)]

Selected other publications: [55, 25, 6, 57, 58, 56, 66, 48, 47]

Highlight: High-level connection between gradient flows and large-deviation principles.

It was realized in 1998 by Felix Otto and co-workers [29] that many well-known partial differential equations are gradient flows. With their introduction of the Wasserstein metric and the accompanying Wasserstein gradient flows, a large number of existing systems were brought together under the gradient-flow umbrella—such as convection-diffusion equations, multiphase-flow equations, the porous media equation, moving-boundary problems, and many others.

However, despite great mathematical interest [5], a *physical* or *mathematical–modelling* understanding of this new structure was still missing.

In a series of papers starting in 2010, Mark Peletier and co-workers built a theory which explains the origin of many gradient-flow structures of well-known PDEs, including the Wasserstein flows and many others. They arise in an intrinsic manner from the large deviations of some underlying, more microscopic stochastic processes.

The most recent work shows that this insight is not at all restricted to Wasserstein gradient flows: in fact, every sequence of reversible stochastic processes satisfying a large-deviation principle provides its deterministic limit with a gradient-flow structure [33]. This insight also gives rise to a natural generalization of Onsager's famous Reciprocity Relations [41, 34].

2.5 Patterns and Waves

The study of patterns and waves has traditionally been a central theme in the Dutch applied mathematics community and lies at the core of the NDNS+ cluster. Spanning myriads of applications, it has continuously been a stimulating and inspiring force for new mathematical challenges and the endeavor to strive for a unifying theory for the most commonly used model equations across disciplines: nonlinear partial differential equations (PDEs). Among the most recent goals within this field are 1) the incorporation of varying coefficients, that is, of non-autonomous/inhomogeneous PDEs both in time and/or in space (rather than the mere tuning of constant parameters) and 2) the analysis of pattern formation in two spatial dimensions.

Top publications: [(xviii),(xix)]

Selected other publications: [27, 10, 50, 31]

Highlight: A model for desertification.

A very fruitful application is the modeling of desertification in ecology, where nonlinear systems of reaction diffusion equations are used to model the degradation of a vegetated area to a desert, the ultimate goal being the elucidation of an effective mechanism to influence or even reverse this process. The systems of nonlinear PDEs arising in this application pose several different mathematical difficulties, such that, to develop analytical tools, it is often more convenient to divert to somewhat simplified versions that still capture the main phenomena. The Klausmeier model and various extensions and generalizations thereof is one such phenomenologically derived model that features a wide range of stable patterns and transient dynamics resembling those observed in nature, while still allowing rigorous mathematical analysis. In a series of articles [13, 59, 49, 12] Arjen Doelman *et al.* investigated the formation of patterns in one and two spatial dimensions revealing the effect of a sloped terrain in form of an additional advection term.

2.6 Stochastic Dynamics

The theme Stochastic Dynamics was introduced to reflect the growing importance of stochastic perturbations in deterministic systems, and more generally the increasing interaction between analysis and stochastics.

The inclusion of stochastic effects in models has become a standard modeling assumption, also where more traditionally mostly deterministic models were considered. Stochastic dynamics also refers to the use of stochastic methods in the study of dynamics, a use that has increased steeply.

Top publications: [(xx)]

Selected other publications: [3, 4, 26, 11, 21, 36, 21, 36, 35, 32, 24, 1, 28]

Highlight: Analysis of stochastic PDEs.

During the past 4 years Jan van Neerven and Mark Veraar (Delft) have worked out several ramifications of their fundamental stochastic maximal regularity result. As in the case in the deterministic setting, maximal regularity enables one to successfully solve various classes of non-linear and/or time dependent stochastic PDEs using fixed point techniques. Along this line of thought, a detailed regularity theory has been established for non-autonomous (stochastic) PDEs with measurable dependence in time. An important development has been to introduce weighted techniques from harmonic analysis. The use of weighted estimates in space and time has enabled them to considerably improve and extend existing results. Among other things, they have shown that the Laplace operator with Dirichlet boundary conditions on a smooth domain has a bounded H^∞-functional calculus on weighted L^p-spaces for very general power weights [64, 22].

2.7 Cluster funding

The primary cluster expenditures are the funding of PhD, Postdoc and Tenure Track positions. These have been listed in the table below.

Table: Funding of NDNS+ by NWO

Names of researchers and universities	Type of research positions	Funded by NWO (in €)	Additional funding	Research focus
Zagaris UT	PhD	100k€	100k€	Multiple Scales
Van Gils UT	PhD	100k€	100k€	Bifurcations and Chaos
Brune UT	TT	240k€	160k€	Variational Methods
Hulshof VU / Prokert TU/e	PhD	100k€	100k€	Multiple Scales
Planqué VU	PhD	100k€	100k€	Patterns and Waves
Van Leeuwen UU	TT	240k€	160k€	Scientific Computing
Homburg/Peters UvA	PhD	90k€	100k€	Bifurcations and Chaos
Batenburg CWI (also DIAMANT)	TT+PhD	345k€	265k€	Scientific Computing
Efstathiou RUG	TT	240k€	160k€	Bifurcations and Chaos
Peletier TU/e	TT+PhD	345k€	265k€	Variational Methods
Van den Berg VU	TT+PhD	345k€	265k€	Bifurcations and Chaos
Chirilus-Bruckner UL	TT	240k€	160k€	Patterns and Waves
Rademacher CWI	PhD	100k€	100k€	Patterns and Waves
Schuttelaars TUD	PhD	100k€	100k€	Multiple Scales
Pop TU/e	Postdoc	100k€		Scientific Computing
Total		2785k€	2135 k€	

Table: Obtained funding under NWO grant schemes

Name of grant scheme	# awards	Amount of NWO funding (in €)	Additional funding, if applicable (in €)
NWO ALW	3	627.495	
NWO Complexity	1	500.000	
NWO Earth and Life Sciences	2	462.872	
NWO ENBARK	1	280.000	
NWO eScience	3	938.000	
NWO ESI-pose	2	1.500.000	
NWO EW	3	663.000	
NWO Free competition	9	1.854.568	
NWO HTSM	1	500.000	
NWO JSTP China	1	300.000	
NWO Mathematics of Planet Earth	7	1.500.000	
NWO NICAS	1	360.000	
NWO NWA Startimpuls	1	2.500.000	
NWO Open Technologie	1	320.000	
NWO Physical Sciences	3	674.135	
NWO Social Sciences	1	200.000	
NWO TOP1	3	1.514.402	
NWO TOP2	3	682.000	
NWO TTW	1	250.000	
NWO URSES	1	650.000	
NWO Visitors Travel Grant	3	23.300	
NWO DST	1	450.000	
NWO CSER	11	2.758.708	
NWO VENI	3	3.250.000	
NWO VIDI	7	4.980.563	
NWO VICI	4	6.000.000	
STW	25	9.820.203	
total		43.187.146	

Table: Obtained funding under international grant schemes

Name of grant scheme	# awards	Amount of funding (in €)	Additional funding
COST Action MP1207 EXTREMA	1	550.000	
CSC-UU (1 PhD student for four years)	3	n.a.	
ERCIM	1	150.000	
EU	7	3.500.000	

European Science Foundation Quan Tissue		7.500	
H2020	3	4.857.000	
INRIA	1	9.000	
Marie Curie European Reintegration Grant	3	395.000	
National Science Foundation Denmark	1	130.000	
NCF Research Grant	2	26.000	
NSF Grow	1	9.000	
Schlumberger foundation	2	96.000	
Turkish Government Grant	1	n.a.	
WAKEUPCALL	1	500.000	
total		10.229.500	

2.8 20 top-publications

Bifurcation and Chaos

(i) J.B. van den Berg and J. Jaquette (2018). A proof of Wright's conjecture. Journal of Differential Equations, 264(12), 7412-7462.

Wright's equation is a classical delay-differential equation with negative feedback. Many have worked towards a proof that for any parameter value above a threshold there is a single slowly oscillating solution that attracts almost all dynamics. This work gives the final missing piece to this long-standing puzzle.

(ii) B. Rink and J. Sanders (2015). *Coupled cell networks: semigroups, Lie algebras and normal forms.* Transactions of the American Mathematical Society, 367(5), 3509-3548.

Dynamical systems like metabolic networks or power grids consist of networks of subsystems that interact and synchronize. This paper connects network dynamics to the representation theory of algebraic semigroups, leading to new tools for computing synchrony patterns and synchrony-breaking bifurcations in networks.

(iii) A. Homburg (2017). Atomic disintegrations for partially hyperbolic diffeomorphisms. Proceedings of the American Mathematical Society, 145(7), 2981-2996.

Shub and Wilkinson showed that volume can be stably ergodic for conservative diffeomorphisms on manifolds. The manifold is invariantly foliated by circles (right). It is shown that volume becomes disintegrated into point measures. Thus 'Fubini's nightmare' naturally occurs in dynamical systems.

(iv) M. Astorg, X. Buff, R. Dujardin, H. Peters, and J. Raissy (2016). A two-dimensional polynomial mapping with a wandering Fatou component. Annals of Math., 263-313.

The non-existence of wandering domains in one complex dimension was conjectured by Fatou (1910's) and proved by Sullivan (1985). This work shows that this celebrated theorem fails in two complex dimensions. The proof relies upon a study of parabolic bifurcations for time-dependent dynamical systems.

Multiple Scales

(v) A. Dubinova, C. Rutjes, U. Ebert, S. Buitink, O. Scholten, and G.T.N. Trinh (2015). Prediction of lightning inception by large ice particles and extensive air showers. Physical Review Letters, 115(1), 015002.

This paper shows a novel explanation of the initiation of lightning due to high-energy cosmic particles, based on discharge simulations combined with rare event analyses of the coincidence of sufficiently high electric fields, sufficiently large ice particles, and a sufficiently strong extensive air shower.

(vi) J.H. van Heerden, M.T. Wortel, F.J. Bruggeman, J.J. Heijnen, Y.J.M. Bollen, R. Planqué, J. Hulshof, T.G. O'Toole, S.A. Wahl, and B. Teusink (2014). *Lost in transition: start-up of glycolysis yields subpopulations of nongrowing cells.* Science, 343(6174), 1245114.

The yeast S. cerevisiae *does not handle transitions from low to high glucose conditions well. This study demonstrates with experiments and mathematical models that yeast glycolysis exhibits bistabilty. It elucidates how a side-branch, trehalose metabolism, ensures that yeast stays in the normal steady state instead of the deadly, imbalanced alternative.*

Scientific Computing

(vii) S. Van Aert, K.J. Batenburg, M.D. Rossell, R. Erni, and G. van Tendeloo (2011). Three-dimensional atomic imaging of crystalline nanoparticles. Nature, 470(7334), 374.

This article demonstrates for the first time that a complex-shaped crystalline nanoparticle can be reconstructed in 3D at atomic resolution from two to four 2D electron microscopy images with a new tomography algorithm that combines combinatorial and numerical mathematical methods.

(viii) B. Sanderse, S.P. van der Pijl, and B. Koren (2011). Review of computational fluid dynamics for wind turbine wake aerodynamics. Wind energy, 14(7), 799-819.

Wind turbine wake aerodynamics is of crucial importance for the design of wind farms. This article reviews computational fluid dynamics techniques for simulating wind turbine wakes, including modeling of the rotor, complex terrain effects, turbulence models, and coupling with atmospheric simulations.

(ix) S. Dubinkina and J. Frank (2010). Statistical relevance of vorticity conservation in the Hamiltonian particle-mesh method. Journal of Computational Physics, 229(7), 2634-2648.

Because of its inherent non-linearity, climate system simulations rely on numerical simulations of the probability distributions, which violate the conservation laws of the continuous system. This paper shows that the Hamiltonian particle-mesh method's conservation of potential vorticity along particle paths improves the probability distribution.

(x) T. van Leeuwen and F.J. Herrmann (2015). A penalty method for PDE-constrained optimization in inverse problems. Inverse Problems, 32(1), 015007.

PDE-constrained optimization is at the heart of many applications in inverse problems, optimal control and parameter estimation. Classical approaches result in a very non-linear optimization problem. This paper analyzes a mildly non-linear alternative approach that approximates the solution of the constrained problem.

(xi) P. van Slingerland, J.K. Ryan, and C. Vuik (2011). Position-dependent smoothness-increasing accuracy-conserving (SIAC) filtering for improving discontinuous Galerkin solutions. SIAM Journal on Scientific Computing, 33(2), 802-825.

In this paper an improved filter is given for enhancing discontinuous Galerkin solutions that easily switches between one-sided postprocessing to handle boundaries or discontinuities and symmetric postprocessing for smooth regions. For the modified postprocessor the error becomes independent of the boundary conditions.

(xii) S. Rhebergen, B. Cockburn, and J.J. van der Vegt (2013). *A space–time discontinuous Galerkin method for the incompressible Navier—Stokes equations*. Journal of Computational Physics, 233, 339-358.

A novel space-time hybridizable discontinuous Galerkin (HDG) finite element discretization of the incompressible Navier–Stokes equations is presented. The numerical discretization preserves higher-order accuracy on time-dependent unstructured meshes and is particularly well suited for free-surface and fluid-structure interaction problems.

Variational Methods

(xiii) A.J. van der Schaft, and B.M. Maschke (2013). Port-Hamiltonian systems on graphs. SIAM Journal on Control and Optimization, 51(2), 906-937.

This paper geometrically models the dynamics of a wide class of physical and biological networks as Hamiltonian systems with energy dissipation. The result implies that the machinery from geometric mechanics is applicable to the analysis and control of these networks, including set-point stabilization and disturbance rejection.

(xiv) K. Efstathiou, and H.W. Broer (2013). Uncovering fractional monodromy. Communications in Mathematical Physics, 324(2), 549-588.

Fractional monodromy is a topological property of singular fibrations with implications for quantum spectra. Appropriate tools for its study have long been elusive. This paper studies fractional monodromy in a general setting, through covering maps and ideas from algebraic topology. This has set the foundations for further results on this problem.

(xv) M. Benning, C. Brune, M. Burger, and J. Müller (2013). Higher-order TV methods—enhancement via Bregman iteration. Journal of Scientific Computing, 54(2-3), 269-310.

This article generalises the BV function and multi-scale theory for the famous variational Rudin-Osher-Fatemi model. For the first time nullspace properties and functional estimates of TGV were proven and Bregman methods paved the way for reconstructing nonlinear higher-order eigenfunctions.

(xvi) S. Adams, N. Dirr, M.A. Peletier, and J. Zimmer. From a large-deviations principle to the Wasserstein gradient flow: A new micro-macro passage. Communications in Mathematical Physics, 307:791–815, 2011.

This paper connects the world of deterministic gradient-flow equations with that of stochastic processes, showing that gradient-flow structures are a natural consequence of large-deviation principles for such processes. This explains the observation that many well-studied partial differential equations are gradient flows.

(xvii) J.H. Evers, S.C. Hille, and A. Muntean (2015). *Mild solutions to a measure-valued mass evolution problem with flux boundary conditions.* Journal of Differential Equations, 259(3), 1068-1097.

The work formulates a mild solution concept for measure-valued solutions to a class of transport equations in a suitable space of measures. The well-posedness of the problem is shown, even in the presence of discontinuous reaction terms. An approximation to this singular case by regular continuous terms is provided.

Patterns and Waves

(xviii) A. Doelman, G. Hayrapetyan, K. Promislow, and B. Wetton (2014). Meander and Pearling of Single-Curvature Bilayer Interfaces in the Functionalized Cahn–Hilliard Equation. SIAM Journal on Mathematical Analysis, 46(6), 3640-3677.

The functionalized Cahn–Hilliard free energy models the appearance and evolution of asymptotically thin morphologies such as membranes in cell biology and synthetic chemistry. This paper addresses the existence and stability of single curvature bilayers and characterize the occurrence and nature of meander and pearling instabilities.

(xix) M. Chirilus-Bruckner, W.P. Düll, and G. Schneider (2014). Validity of the KdV equation for the modulation of periodic traveling waves in the NLS equation. Journal of Mathematical Analysis and Applications, 414(1), 166-175.

This work gives a new analytical approach along periodic traveling waves in the Nonlinear Schrödinger (NLS) equation. A novel choice of coordinates, function spaces and estimation techniques reduces the method's technicality. Modulations of amplitude and phase in the NLS can be described by the Korteweg–de Vries equation on long time scales.

Stochastic Dynamics

(xx) J.M.A.M. van Neerven, M.C. Veraar, and L. Weis (2012). Stochastic maximal L^p-regularity. Ann. Probab. 40 (2012), 788-812.

In this paper maximal regularity for stochastic partial differential equations is proved, which solves a long standing open problem. Consequently, many difficult semi-linear, quasilinear and non-autonomous stochastic partial differential equations can be solved by a fixed point argument.

3 Participation Dutch Mathematics Community

The NDNS+ cluster brings together all mathematicians in the Netherlands who study the dynamical behavior of natural systems from a modeling, numerical or analytical point of view. At present this involves researchers from the three technical universities (TUD, TU/e, UT), five general universities (UU, UvA, RUL, RUG, VUA), and the national research center CWI. The activities of the cluster contribute to community building, stimulate the exchange of research expertise (both national and international), disseminate the research done within NDNS+, the creation of a stimulating environment for PhD students and innovation of research.

Through so called ‘advisor positions’, collaboration between the NDNS+-institutes has been stimulated. Through small grants (10k€), it has been made possible to reduce teaching load, facilitating to spend time at another institute. In 2011, 7 advisor positions have been financed.

There is ongoing collaboration with the GQT cluster. Since 2014, a biennial international mini-workshop on Symplectic Geometry is jointly organized. The GQT cluster supported the appointment of a NDNS+-tenure track position in topological data analysis.

Computational imaging forms a bridge between our cluster and DIAMANT, resulting in collaboration between UU (Tristan van Leeuwen), UT (Christoph Brune) and CWI (Joost Batenburg). The work of Mark Peletier on generalized gradient flows and large deviations is on the interface with STAR. We foresee that interactions with STAR will occur more frequently, especially within the themes ’dynamics and data’ and ’dynamical systems theory’.

4 Activities, participation target groups and expenditures (facts and figures)

The aim of the NDNS+ cluster is to create a community of PhD students, postdocs and staff from the cluster to share ideas and to promote research. To achieve this goal, we use several instruments: the support for workshops helps NDNS+ researchers to organize meetings; the PhD Days create a community of PhD students where also interesting lectures by staff members are given; advisor positions stimulate collaborations across different institutions; travel grants allow PhD students to make long-term visits to foreign institutions, thereby expanding their network and broadening their viewpoint. The NDNS+ meetings (two days) are meant for community building and exchange of ideas. In 2019 we will combine this activity with the yearly meeting of the Dutch Mathematical Society (NMC). As all the clusters will do so, this will be a stimulus for the NMC.

Table: Activities and budget

Activity	Budget	Aim	Target group	Number	Decision making procedure
Support for workshops and conferences	143k€	Stimulating vibrant research climate; strengthening (inter)national ties	All researchers in NDNS+ field	70 workshops & Conferences	NDNS+ board
NDNS+ PhD travel grants	71k€	Creating opportunities for talented students	PhD students	20 international research visits of 1–3 months	NDNS+ director
Advisor positions (staff exchange)	70k€	Enhancing collaboration between the NDNS+ research groups	faculty of NDNS+ research groups	7 exchanges	NDNS+ board
NDNS+ two-day meetings	38k€	Enhancing collaboration between the NDNS+ research groups; community building	National NDNS+ research community	4; 30–50 participants (2015–2018)	NDNS+ board

NDNS+ PhD Days	34k€	Community building for PhD students and broadening their perspective	PhD students	7; 30–40 participants (2012–2018)	NDNS+ board
summer schools	10k€	Offering opportunities for PhD students to broaden and deepen their knowledge	PhD students	3	NDNS+ board
International visitors (1–3 months)	19k€	Stimulating international collaboration, stimulating research climate	Senior researchers	5	NDNS+ board
local activities	32k€	Stimulating activity in the NDNS+ research field	All researchers in NDNS+ field	Many small ones	Local NDNS+ team leader
PWN-Deloitte report (contribution)	10k€	Exhibiting the importance of mathematics for the Dutch economy	Decision makers	1	NDNS+ board
MasterMath	10k€	Offering an excellent national Master programme in dynamical systems and analysis	Master/PhD students	5–8 courses per year	NDNS+ board
Total	437k€				

5 Relevance of the cluster including knowledge transfer

Through the cluster, in total *9 new tenure track positions* have been realized. In the 2013 round Christoph Brune (UT), Tristan van Leeuwen (UU), Konstantinos Efstathiou (RUG) and Martina Chirilus-Bruckner (RUL) were appointed. Two more candidates in this round got a tenure track position, although not financed by the cluster: Svetlana Dubinkina (CWI) and Oliver Fabert (VUA). In 2017, three more tenure track positions were obtained: Magnus Botnan (VUA), Pierre Nyquist (TU/e) and Felix Lucka (CWI). The tenure trackers from the 2013 round have a significant impact in the mathematical community in the Netherlands.

Within the national mathematics teaching program Mastermath Christoph Brune and Tristan van Leeuwen jointly developed the course 'Inverse problems in Imaging', Konstantinos Efstathiou is responsible for 'Advanced Hamiltonian Mechanics' (which is a joint GQT, NDNS+ course) and Martina Chirilus-Bruckner is one of the teachers of the course 'Nonlinear Waves'.

During the period under consideration, 158 PhD students were active. In the following table we list the outflow according to activities after finishing.

Consultancy	Industry	Postdoc	Education	Publisher	Unknown	Not yet finished
6	25	25	20	1	30	51

5.1 Collaboration with other disciplines

It is in the DNA of the NDNS+ cluster to build strong relationships with other disciplines, like biology, chemistry, ecology, economy, and physics. Here we give examples of collaboration with physics and the life sciences.

physics

A nice example of the close interaction between mathematics and physics in NDNS+ is the collaboration between the Mathematics of Computational Science group (Department of Applied Mathematics) and the Complex Photonic Systems group (Department of Applied Physics) of the University of Twente. Both groups study ways to manipulate light at the nanoscale using photonic crystals and collaborate in four NWO funded PhD projects. This nanophotonics research requires both the development, analysis and implementation of novel discontinuous Galerkin eigenvalue solvers for the time-harmonic Maxwell equations and their implementation in software running efficiently on parallel clusters. A highlight in this research is the recent

JCER project "Accurate and Efficient Computation of the Optical Properties of Nanostructures for Improved Photovoltaics", in which novel Discontinuous Galerkin Virtual Element Methods (DGVEM) are developed for the computation of light in real fabricated 3D photonic crystals whose geometry is measured by the COPS group in the European Synchrotron Radiation Facility using X-ray holotomography techniques. Until now only computations on idealized structures were possible in nanophotonics. The very large data sets resulting from the X-ray experiments contain, however, rough surfaces that require a whole new approach to discretize the Maxwell equations using DGVEM on polyhedral non-convex elements. Standard finite elements methods would simply not be efficient for these problems due to the very large geometric complexity.

In Groningen, Holger Waalkens is studying classical and quantum transport in Hamiltonian systems. Of special interest is the flux through bottlenecks linking different phase space regions that represent metastable states. This setting is very general with applications reaching from the conformational changes of a molecule to the capture of asteroids in the solar system. It can be shown that the transport is mediated by normally hyperbolic invariant manifold and their stable and unstable manifolds which on a microscopic scale also form the skeleton for quantum transport. In collaboration with physicists and chemists he is working on the computation of the invariant manifolds in multi-degree-of-freedom Hamiltonian systems and their utilization for determining classical and quantum transport rates.

life sciences

Life Sciences are becoming an ever more important application area for NDNS+ and are a rich inspiration for dynamical systems theory, scientific computing, and multiscale systems. Organizationally, NDNS+ members are active in the national Life Sciences community. For instance, Roeland Merks is member of the steering committee of the Origins Center, a national initiative funded by the Dutch National Science Agenda.

Recent contributions of NDNS+-scientists have led to new insights in life sciences as well as new life-science-inspired mathematics, often in collaboration with experimental biologists. At the ecological scale, in close collaboration with ecologists, Arjen Doelman and coworkers have analyzed sets of advection-reaction-diffusion equations to explain pattern formation in water-limited (arid) ecosystems [53], and they have uncovered dynamics according to the Cahn-Hilliard equations in mussel beds [31]. At the cellular and tissue scale, Fred Vermolen and Kees Vuik develop partial-differential equation and Lagrangian cell-based models of wound healing in collaboration with bio-engineers, with the ultimate aim to identify new strategies to prevent scar formation [30] and assist treatment planning [67]. Roeland Merks *et al.* develop hybrid, multi-scale cellular Potts and partial-differential equation-based simulation models to uncover how the chemical and mechanical interaction between cells and their micro-environment drive blood vessel formation [65] and cell alignment in tissues [42]. Also, the work on neural networks in the team of Stephan van Gils (see highlight multiple scales) is a good example of cell-to-tissue scale modeling in collaboration with experimentalists. At the molecular level, using ODE modeling the Bob Planqué and Joost Hulshof teams have predicted that glucose metabolism in yeast has an alternative steady state, due to imbalance of phosphate. The prediction was confirmed experimentally and has resulted in a publication in *Science* [63].

5.2 Collaboration with Dutch R&D institutions and/or industry

Collaboration with the Dutch R&D institutes and industry is mainly developed through STW programs. Based on the programs that were funded in the period 2010-2018 we list the following companies and institutes, which were part of the users-committee for at least one of the programs.

Table: Companies involved in STW programs

Company/Institute	Company size <> 250	url
ABB Corporate Research Center, Switzerland	> 250	https://new.abb.com/about
Additive Industries	< 250	https://additiveindustries.com
Alliander	> 250	https://www.alliander.com/en
Aluchemie	> 250	https://www.aluchemie.nl/en/home.html
Almatis	> 250	http://www.almatis.com
ASML	> 250	https://www.asml.com/
Bekaert Combustion Technology B.V. (BCT)	> 250	https://www.bekaert.com/en/
Bosch Thermotechniek B.V.	> 250	https://www.bosch-industrial.nl
Controllab Products B.V.	< 250	http://www.controllab.nl
Damen shipyards	> 250	https://www.damen.com/en?
DEMCON	> 250	https://www.demcon.nl/en/
DNV GL – Energy, KEMA Labs in Arnhem	> 250	https://www.dnvgl.com/energy/
DNV-GL (Groningen)	> 250	https://www.dnvgl.com
Dutch Institute for Fundamental Energy Research	< 250	https://www.differ.nl
European Space Agency (ESA)	> 250	https://www.esa.int/ESA
HZPC	> 250	https://www.hzpc.com
Infinite Simulation Systems	< 250	https://www.infinite.nl/bedrijf/contact/
Ipsum Energy	< 250	https://www.ipsumenergy.com/en/
Micro Turbine Technology B.V. (MTT)	< 250	https://www.mtt-eu.com
MI partners	< 250	http://www.mi-partners.nl
Netherlands Institute for Space Research	> 250	https://www.sron.nl
Plasma Pendix	< 250	
Royal Boskalis Westminster N.V.	> 250	https://boskalis.com
Shell	> 250	https://www.shell.com
Stichting FloodControl IJkdijk	< 250	http://floodcontrolijkdijk.nl/en/
Sympower	< 250	https://www.sympower.net/about/our-story
Tata steel	> 250	http://www.tata.com
Tennet (Arnhem)	> 250	http://karriere.tennet.eu
TNO	> 250	https://www.tno.nl/en/
Van Oord	> 250	https://www.vanoord.com
Westnetz GmbH	> 250	https://iam.westnetz.de

5.3 Outreach activities

Table: Outreach activities

year	activity
2011	Article in ‘de Volkskrant’ (national newspaper) about Ute Ebert’s research on sprites.
2011	Video ‘De wereld is wiskunde’ on Mark Peletier’s work as mathematician
2012	Video ‘Forging Steel with Mathematics’/‘Staal smeden met wiskunde’ on Lucia Scardia’s work that combines maths with engineering to provide insight into old problems in plasticity
2012	Video ‘Adjusting the Rudder’/‘Het roer aanpassen’ on David Bourne’s contribution to the Study Group Mathematics with Industry 2011
2012	Roeland Merks in the opinion magazine ’De Groene Amsterdammer’ on computer models in biology.
2012	Public Radio interview with Jan Bouwe van den Berg about mathematical modeling
2012	Public Radio Interview with jan Bouwe van den Berg on complex patterns
2013	Arjen Doelman and coworkers in ‘de Volkskrant’ on the movement of mussels.
2013	Radio Broadcast on the prediction of traffic jams.
2014	Barry Koren in DWDD (a very popular talk show at prime time.)
2014	Jan Bouwe van den Berg in ’Wervelende wiskunde en superhelden’ youtube movie of this event
2015	Giovanni Bonaschi in Video on the paper ’Quadratic and Rate-Independent Limits for a Large-Deviations Functional’
2015	Ute Eberts lab in the news with research on thunderstorms: New York Times, Frankfurter Algemeine Zeitung, The Daily Mail
2015	Ute Ebert in TV broadcast about thunderstorms (’Biblic Plagues’)
2015	Kees Vuik in ’The Guardian’ on Prediction of earthquakes
2016	Eric Siero in a radio interview on desertification.
2017	4TU.AMI Spring Congress 2017: Mathematics for health
2017	National Mathematics Symposium for Dutch mathematics students

2017	Danil Koppenol in Radio program on mathematics for wound healing
2017	Fred Vermolen and Danil Koppenol on mathematics for wound healing in National Education Guide
2018	Christoph Brune in the news with 4TU Precision Medicine Program
2018	Sjoerd Rienstra receives the CEAS Aeroacoustics Award 2018
2018	Paolo Cifani receives the Wim Nieuwpoort Award for research into bubbly turbulence.

Part B: Roadmap

6 Organization and management of the cluster

The cluster brings together research groups from eight universities and CWI; the groups and their members are listed in Appendix A. Each of these groups is associated with several of the research themes outlined above, and there already exist many personal and scientific connections between the groups. The cluster is open for every person/group that wants to join. This resulted recently in a ninth university, (Radboud University), that will join.

The cluster is managed by a board, which presently consists of the eight members listed in section 1. The main task of the board is to coordinate the scientific activities of the cluster regarding the organization of workshops, visiting positions, joint research, joint seminars. In the coming years, special attention will be given to the initiation of large projects (for instance at the European scale or the Dutch Scientific Agenda). There is secretarial support for NDNS+ at the University of Twente, where also the website is hosted and maintained. The board meets twice a year and irregularly when important decisions are on the agenda.

6.1 New themes

Current research of NDNS+ focuses on *Bifurcation and Chaos, Multiple Scales, Scientific Computing, Variational Methods, Patterns and Waves*, and *Stochastic Dynamics*. Shifts in the national and international research landscape require that we critically reflect on these themes. The overall focus of the NDNS+ cluster will remain in the area of Nonlinear Dynamics. After careful evaluation, we have identified the following themes for the coming decade:

1. Dynamical Systems Theory
2. Computational Dynamics
3. Dynamics and Data
4. Emergence

6.1.1 Dynamical Systems Theory

Dynamical systems arise as mathematical models throughout the sciences. They describe the evolution of quantities that change in the course of time. These models can be as simple as a linear differential equation for the growth of a bacterial colony or an electronic circuit, and as difficult as the stochastic partial differential equations arising from statistical mechanics and quantum field theory. Differential equations of various degrees of complexity are nowadays used, for example, to compute the spreading of diseases, to predict the weather and climate, to determine the fate of species under evolutionary competition, to model economic markets, to calculate the orbit of a satellite under gravitational attraction, etc. Less standard dynamical systems such as iterated function systems, data driven and equation free models, and hybrid systems are meanwhile becoming more important in applications.

Dynamical systems are also a key tool for answering fundamental questions inside mathematics. For example, they have played a pivotal role in number theory (Green and Tao used dynamics to prove their famous result on arithmetic progressions), topology (a so-called geometric flow was the crucial element of Perelman's proof of the Poincaré conjecture), and complex and symplectic geometry (Teichmuller flow; Floer theory). Moreover, ideas and language of dynamical systems theory have drastically changed our view on partial differential equations and mathematical physics in recent years. Illustrative in this respect is the fact that dynamical systems play an essential role in the work of 13 of the 26 mathematicians who received a Fields medal between 1994 and 2018.

Dynamical systems research in the Netherlands has been world-leading since the 1970s. It originally focused on the foundations of the subject, in particular chaos theory, bifurcation theory, and asymptotic analysis. Current research within NDNS+ on, for example, applications of dynamical systems in mathematical biology and earth and climate science, grew directly out of this tradition. Maintaining and strengthening the leading position of our country in this field requires a continuous and significant investment in fundamental research of dynamical systems. In the coming years, the NDNS+ research team will focus on

1. inventing topological and variational techniques for computing orbits of iterated maps and differential equations, making use, for example, of set-valued maps and generalized Morse and Floer homology;
2. better understanding the stochastic properties of deterministic systems, including random maps, and their ergodicity, intermittency, and limit theorems;
3. describing the dynamics and bifurcation scenarios of systems with network structure, in particular the question how and why networks synchronize. This also entails developing tailored computational tools for network dynamical systems;
4. the rigorous justification of dimension reduction techniques, including homogenization, and amplitude, modulation, and mean field equations;
5. designing computational techniques for the rigorous validation of solutions to high dimensional ODEs as well as PDEs.
6. developing analytic and computational tools for studying the dynamics of stochastic and geometric PDEs, which describe complex dynamic phenomena in the earth and life sciences.

6.1.2 Computational Dynamics

From the Dynamical Systems Theory theme it is manifest that there are many applications of dynamical systems in physics, biology, finance, to name only a few. It is essential to have theoretical results concerning the solution (existence, uniqueness, approximation), but it is not always possible to answer these questions for more complex systems or to obtain sufficient insight in the shape of the solutions by analytic means. The combination of theory and numerical methods is of great importance for such problems. Indeed, dynamical systems can be very difficult to solve with standard numerical methods for several reasons: bifurcations can occur, the problem may have multiple time scales, ill-posedness in the neighborhood of singularities, etc. In developing new numerical approaches to solve relevant dynamical systems these properties should be taken into account. Starting from "simple" problems for which an analytical expression for the solution is known, the numerical approximations can be compared with the known solutions. This gives confidence that the methods can also be used for dynamical systems where the solution is not explicitly known, especially when complemented by convergence results for the numerical scheme.

Many dynamical systems that are pivotal in applications have special properties, such as symmetries, conservation properties, positiveness, etc. It is very important to develop numerical methods which lead to approximations that also have these properties (sometimes only in a discrete sense). Numerical methods which preserve such properties are known as structure-preserving methods. Furthermore, characteristic for many dynamical systems is the multi-scale character, which means that large and small time scales (which can differ by many orders of magnitude) are involved. Straightforward numerical methods can not be used to solve such systems due to excessive computing times. This motivates to look for special numerical methods which can handle multi-scale problems or to reformulate the problem such that the multi-scale character is taken into account, while the resulting system is solvable with standard numerical methods. Moreover, bifurcations occur in many problems ranging from predator prey models to the flow of the Gulf Stream (a warm and swift Atlantic ocean current). Good approximation of the solution near bifurcations is indispensable since the consequences of missing a bifurcation can be paramount. In general this problem is ill posed (unstable behavior) close to a bifurcation. This makes the development of a robust numerical method very challenging.

There is a long tradition in the Netherlands of developing advanced numerical methods for large dynamical systems. Research has been done to develop Computational Fluid Dynamics methods which can be used for climate and weather prediction. Additionally, numerical methods are constructed to compute pattern formation in biological and geoscience applications. Computational dynamics methods are also developed for multi-body systems and stochastic applications.

In the coming years, the NDNS+ research team will focus on

1. Developing advanced structure-preserving numerical discretizations that can for example be used to solve the three dimensional incompressible Navier–Stokes equations for weather prediction and ocean flows.

2. Develop methods for multi-scale and stochastic problems, for example to deal with evolving patterns in biology and coastal regions (coupling water flow and sediment transport). Multi-scale (time and space) behavior is crucial for these problems.

3. Proving that the developed algorithms are reliable, through numerical analysis, approximation theory, convergence results and computer-assisted proof techniques. This may not always be feasible in all generality, but can be accomplished for simplified configurations.

4. Developing scalable methods for imaging. High resolution of the earth or medical images are of primary importance. Current methods are not able to approximate these images with the required accuracy within a reasonable amount of work.

5. Since many problems lead to large simulations, the methods should be formulated in such a way that they are suitable for modern high performance computing hardware. Typical hardware examples are: large parallel clusters, multi-GPU machines, and FPGA's.

6.1.3 Dynamical Systems & Data

The ability to discover physical laws and governing dynamical systems equations from data is one of humankind's greatest intellectual achievements. To study such temporal evolution of natural processes, the traditional dynamical systems approach is to formulate a specific mathematical model, often in the form of a (highly simplified) system of differential equations, which is subsequently analyzed by theoretical and/or numerical means and compared to actual observations. In practice however, this procedure is often frustrated by noisy measurements, crucial variables that cannot be easily observed or parameters that cannot be estimated reliably. The automatic discovery of hidden parameters or functions in dynamical systems from observed data is an ill-posed inverse problem (data assimilation). Although, classical regularization theory and methods exist, the automatic learning of very sensitive parameters, significantly influencing the bifurcation analysis near chaotic regimes is still extremely challenging.

In recent years, there has been a first push towards developing data-driven methods to address these challenges. Dynamical systems developed in theoretical neuroscience and deep recurrent neural networks applied in data science share a lot of similarities. These methods often bypass the need to have exact model equations, but still leverage our understanding of the main qualitative features that dynamical systems can exhibit.

In the coming years, the NDNS+ research team will focus on the following five key topics to address interface between dynamics and data via a unique combination of inverse problems, dynamical systems and machine learning theory:

1. *Inverse problems & data assimilation.* Developing and analyzing variational methods and nonlinear, nonlocal regularization techniques for inverse problems subject to integro-differential equations, reflecting basic neural networks, is the first step. It connects to data assimilation which has greatly been influenced by dynamical systems.
2. *Koopman operators for spectral analysis.* The main idea behind Koopman theory is that the action of dynamical systems can be lifted from the space of physical variables to the set of possible measurements on the physical system. The Koopman operators describe the way in which these measurements evolve under the action of the dynamical system. A systematic comparison to spectral methods of nonlinear gradient flows regarding homogenous functionals, e.g. of low-rank type, could offer great new insights.
3. *Persistent homology & optimal transport on graphs.* When viewed as a collection of points in some possibly high-dimensional space, the shape and clusters of a dataset often reflects important patterns within the data. Higher-dimensional holes or voids may correspond to impossible or unstable configurations (bifurcation analysis). Persistent homology condenses the geometric information down into a useable format, while also retaining and highlighting important nonlinear and global features of the dataset's shape. This promising area is related to Wasserstein transport and clustering methods on graphs.
4. *Discovery of equations & model sparsity.* It is a major challenge to turn raw data into models that are not just predictive (data assimiliation), but also provide insight into the nature of the underlying dynamical system that generated the data. Furthermore, brute force data assimilation may lead to overfitting and high-dimensional models with little predictive power. Dimensionality reduction via quantification of relative importance of state variables and reduced basis methods will be of much interest.
5. *Deep machine learning for PDEs.* Deep recurrent artificial neural networks share a lot of structural and numerical similarities with nonlinear dynamical systems and are able to represent complex nonlinear, multiscale maps in a compressible linearizable fashion. Whereas nearly all past approximation algorithms for PDEs suffered from the well-known curse of

dimensionality, deep learning networks are able to tackle this dimensionality reduction efficiently via automatically learned emerging multiscale patterns (wavelet scattering and hierarchical low-rank structures).

6.1.4 Emergence

The interaction of a large number of entities, each of which behaves according to rather simple but nonlinear rules, often leads to rich, complex and unexpected collective behavior. The building blocks appear to act in unison, displaying so-called emergent behavior. We see such emergent dynamics in cars driving on a busy road, in neural networks in the brain, in atomic lattices in a composite material, and in the molecules in the atmosphere and oceans forming the weather system. Indeed, emergent phenomena manifest themselves on all possible scales, from particle physics to the cosmos. At each new scale, the central question is: which pattern emerges, and which rules govern its dynamics?

The mathematics of dynamical systems is pivotal in describing and predicting complex emergent phenomena. One challenge is the continuum behavior of interacting particle systems, which involves both PDE theory and large deviation principles. Another important area is the study of pattern formation, which has an enormous diversity of applications: from desertification to phase transitions, from structure selection in block copolymers to the dynamics of sandbanks in the Waddenzee. Yet another manifestation is collective behavior on networks: synchronization, rhythmic oscillation and structured chaos in networks ranging from metabolic pathways to power grids. More generally, central to the theory of dynamical systems are (symmetry breaking) bifurcations, which are universal scenarios that play a fundamental organizing role in emergence. These 'normal forms' are powerful tools for making qualitative and quantitative predictions about complex emergent phenomena.

Emergent behavior often occurs in the form of some coherent structure: a shock wave in the traffic jam, a traveling signal in the brain, plastic behavior in materials due to localized defects, an eddy or cyclone in the atmosphere. The dynamics of the interacting building blocks can often be studied through the analysis of singular limits, based on multiscale techniques. This usually reveals an emergent low(er) dimensional invariant attractor, on which new laws of motion (which may be local, nonlocal or a combination) can then be derived.

To make progress on such problems we need to develop novel mathematics as well as new numerical algorithms. One cannot rely on scientific computing alone, since the size of the system is often simply many orders of magnitude too big to keep track of in a computer calculation. Effective computational methods for simulating large numbers of interacting units could take advantage of the analysis of the limit where the number of interacting particles tends to infinity. And in the opposite direction, discretizing the continuum limit leads to other deep mathematical questions, such as: how fine-grained does the discretization need to be so that it reliably conveys the behavior of the continuum model?

Some of the central challenges are:

1. How can we analyze systems where the emergent pattern feeds back to influence the behavior of the individual units? In particular, when does this feedback ("downward causation") lead to stabilization? Notorious examples are multicellular biological systems, e.g., the heart. Here ion currents drive the propagation of excitation waves; these induce tissue contraction, the resultant mechanical strains again open ion channels, locally changing the ion currents.
2. How can we control emergent behavior on networks (synchronization, rhythmic oscilla-

tion, structured chaos) ranging from metabolic pathways to power grids? Can one design networks which exhibit certain desired collective behavior?

3. In multi-scale modeling: can we reliably include scales that are not explicitly modeled, e.g. through stochastic components or through averaging/homogenization techniques, and how to incorporate this efficiently in simulations?
4. When can we be sure that a (numerical) discretization is sufficiently fine-grained to capture the true (emergent) behavior of the continuum system?
5. How can we understand which pattern emerges (is selected) in materials where there are many stationary states, all far from homogeneous?
6. Can we understand the emergence of plastic behavior in materials (upscaling dislocations)?

6.2 Development of research staff

Since the beginning of this century, the inflow of students in mathematics at Dutch universities has increased from below two hundred, to over one thousand in 2015 and is still increasing. We see a similar increase in other beta-sciences, leading to a strong increase in service teaching, especially at the technical universities. This puts enormous pressure on the research time of the staff. Although the hiring of tenure trackers has alleviated the pressure a little bit, we *need at least 10 fte* in the next decade in this cluster to maintain our research capacity at the same level. We hope that these positions can be realized through the sector plan for mathematics.

In coordination with other clusters, we came to the following proposal: our priority for 2019 is a national call of 4 M€, focusing on the 7 broad research themes envisioned in the sector plan, to be spent on PhD positions, both matched and unmatched. It is our wish that the clusters themselves will form the evaluation committee, ensuring an even division of positions between the clusters, while for a specific theme all clusters linked to that theme will evaluate in collaboration.

In the table below we list the a number of yearly expenses. Most items speak for themselves. Note that we require funding to create *research exchange positions* to facilitate exchange between the nodes within the cluster, but also across clusters. On the topic 'Dynamical Systems & Data' strong collaboration with STAR is necessary, and possible, to advance the subject. The money involved is meant to reduce the teaching load for the one who travels.

Additional funding is necessary for the managing of the cluster. This report could not have been made without the help of the secretaries at UT. Maintaining a website will increase the coherence in the cluster, but clearly requires a time investment. We foresee more coordinating tasks for the acquisition of larger grants within the Dutch Science Agenda or in Europe.

The largest amount is reserved for PhD positions, which are crucial to stimulate the research as described in our roadmap. Through matching money, we will try to actually appoint more than the four PhD positions mentioned in the table.

Table: Requested funding

budget item	target group	goal	costs (k€/yr)
Workshops/conferences	NDNS+ researchser	Research collaboration	20
PhD days	NDNS+ PhD's	Community building	4
NDNS+ activities NMC	NDNS+ researchers	Community buidling	8

Outreach activities	Society	Communication NDNS+ activities	4
Focused activity groups	NDNS+ researchers	Stimulation focused activities (call preparation)	20
PhD travel grants	NDNS+ PhD's	International visits	30
International long-term visitors	Foreign visitors in NDNS+ area	Research collaboration	15
International visits	NDNS+ researchers	International collaboration	15
Research exchange	NDNS+ members	Stimulate joint research	20
Secretarial support	Managing director	Support of the board	10
4 PhD-positions	NDNS+ researchers	Research stimulus	800
total			**946**

7 SWOT Analysis

Table: SWOT Analysis

Strengths	Weaknesses	Opportunities	Threats
Within the area of dynamical system we cover the full range from fundamental to very applied research	Not yet able to contribute as a cluster to large grant proposals	The cluster has the potential to contribute to societal problems, which can also be a significant source of mathematical research questions and funding	There are not enough possibilities for funding of the more fundamental research
Research is of high quality according to research grants, collaboration with industry, visibility in media, publications in top journals	Relatively low inflow of students in Mastermath courses dedicated to NDNS+ research topics	The diversity in specialization within the cluster makes the cluster interesting for large collaborations	There is too much pressure on research due to an increasing teaching load
Connection between research and eduction is optimal through Mastermath and teaching programs at our universities	Scientific Computing is not yet fully integrated in the cluster	Strengthening ties with the STAR cluster will be mutually beneficial, especially in the Dynamics and Data theme	
We are able to attract excellent PhD students and tenure trackers			

7.1 Strategy for NDNS+ motivated by the SWOT

The main aim of the NDNS+ cluster remains to stimulate the high quality research performed by the dynamical systems community in the Netherlands. The full breadth of the field, from fundamental to applied, is supported by focussing on our four newly formulated themes. The importance and visibility of scientific computing is highlighted by the computational dynamics theme. There are tremendous opportunities in the theme of dynamics and data for fruitful collaboration with the STAR cluster. More generally, cooperation with the other clusters on the *sectorplan* themes is envisaged.

The NDNS+ cluster will nourish its crucial role in community building, organising and supporting international research activities in dynamical systems, as well as stimulating young researchers.

We intend to initiate the formation of consortia for larger grant proposals. The NWA calls form excellent opportunities for funding of NDNS+ research. The cluster is well positioned to make seminal contributions to the solution of societal problems, ranging from climate change to personalised medicine to the energy transition. In this light it is particularly natural to join forces with STAR (and possibly other clusters) to make such consortia even more attractive and influential. Participation of the cluster in a European consortium should also be investigated.

The increasing number of bachelor students will likely lead to a natural increase of students in NDNS+ courses. But we will also take an active approach towards making our MasterMath offering more attractive, by developing courses on contemporary topics linked to current research, as well as offering courses in collaboration with other clusters which appeal to a broader group of master students.

The threat of the lack of funding for fundamental research is ever-present, but we are optimistic that the *sectorplan* will alleviate the most pressing issues. This will lead to a better balance between research and teaching. It will also allow NDNS+ members to invest time in various fora to show that interdisciplinary research is key for mathematics to remain a healthy, valuable and fascinating discipline.

References

[1] N. Abbasi, M. Gharaei, and A.J. Homburg. Iterated function systems of logistic maps: synchronization and intermittency. *Nonlinearity*, 31(8):3880, 2018.

[2] A. Al-Hdaibat, W. Govaerts, Y. Kuznetsov, and H. Meijer. Initialization of homoclinic solutions near Bogdanov–Takens points: Lindstedt–Poincaré compared with regular perturbation method. *SIAM Journal on Applied Dynamical Systems*, 15(2):952–980, 2016.

[3] T. Alkurdi, S.C. Hille, and O. van Gaans. Ergodicity and stability of a dynamical system perturbed by impulsive random interventions. *J. Math. Anal. Appl.*, 407(2):480–494, 2013.

[4] T. Alkurdi, S.C. Hille, and O. van Gaans. Persistence of stability for equilibria of map iterations in Banach spaces under small random perturbations. *Potential Anal.*, 42(1):175–201, 2015.

[5] L. Ambrosio, N. Gigli, and G. Savaré. *Gradient Flows in Metric Spaces and in the Space of Probability Measures*. Lectures in Mathematics ETH Zürich. Birkhäuser, 2005.

[6] B. Bakker, J.B. van den Berg, and R. Vandervorst. A Floer homology approach to travelling waves in reaction-diffusion equations on cylinders. *arXiv preprint arXiv:1702.00772*, 2017.

[7] R. Beltman, M.J.H. Anthonissen, and B. Koren. Conservative polytopal mimetic discretization of the incompressible Navier–Stokes equations. *Journal of Computational and Applied Mathematics*, 340:443–473, 2018.

[8] H.W. Broer and H. Hanßmann. On jupiter and his galilean satellites: Librations of de sitter's periodic motions. *Indagationes Mathematicae*, 27(5):1305–1336, 2016.

[9] A. Buhr and K. Smetana. Randomized local model order reduction. *arXiv preprint arXiv:1706.09179*, 2017.

[10] M. Chirilus-Bruckner, A. Doelman, P. van Heijster, and J.D.M. Rademacher. Butterfly catastrophe for fronts in a three-component reaction–diffusion system. *Journal of Nonlinear Science*, 25(1):87–129, 2015.

[11] D. Crommelin and E. Vanden-Eijnden. Diffusion estimation from multiscale data by operator eigenpairs. *Multiscale Model. Simul.*, 9(4):1588–1623, 2011.

[12] A. Doelman, J.D.M. Rademacher, B. de Rijk, and F. Veerman. Destabilization mechanisms of periodic pulse patterns near a homoclinic limit. *SIAM Journal on Applied Dynamical Systems*, 17(2):1833–1890, 2018.

[13] A. Doelman, J.D.M. Rademacher, and S. van der Stelt. Hopf dances near the tips of Busse balloons. *Discrete and Continuous Dynamical Systems - S*, 5(1):61–92, 2012.

[14] A. Dubinova, C. Rutjes, U. Ebert, S. Buitink, O. Scholten, and G.T.N. Trinh. Prediction of lightning inception by large ice particles and extensive air showers. *Physical review letters*, 115(1):015002, 2015.

[15] U. Ebert, F. Brau, G. Derks, W. Hundsdorfer, C. Kao, C. Li, A. Luque, B. Meulenbroek, S. Nijdam, V. Ratushnaya, et al. Multiple scales in streamer discharges, with an emphasis on moving boundary approximations. *Nonlinearity*, 24(1):C1, 2010.

[16] K. Efstathiou and H. W. Broer. Uncovering fractional monodromy. *Commun. Math. Phys.*, 324(2):549–588, 2013.

[17] T.L. Eissa, K. Dijkstra, C. Brune, R.G. Emerson, M.J.A.M. van Putten, R.R. Goodman, G.M. McKhann, C.A Schevon, W. van Drongelen, and S.A. van Gils. Cross-scale effects of neural interactions during human neocortical seizure activity. *Proceedings of the National Academy of Sciences*, 114(40):10761–10766, 2017.

[18] Y.A. Erlangga and R. Nabben. On a multilevel Krylov method for the Helmholtz equation preconditioned by shifted Laplacian. *Electronic Transactions on Numerical Analysis*, 31:403–424, 2008.

[19] Y.A. Erlangga, C.W. Oosterlee, and C. Vuik. A novel multigrid based preconditioner for heterogeneous Helmholtz problems. *SIAM J. Sci. Comput.*, 27:1471–1492, 2006.

[20] O. Fabert. Gravitational descendants in symplectic field theory. *Commun. Math. Phys.*, 302(1):113–159, 2011.

[21] J. Frank, B. Leimkuhler, and K.W. Myerscough. Direct control of the small-scale energy balance in two-dimensional fluid dynamics. *Journal of Fluid Mechanics*, 782:240–259, 2015.

[22] C. Gallarati and M. Veraar. Maximal regularity for non-autonomous equations with measurable dependence on time. *Potential Anal.*, 46(3):527–567, 2017.

[23] S. Geevers and J.J.W. van der Vegt. Sharp penalty term and time step bounds for the interior penalty discontinuous galerkin method for linear hyperbolic problems. *SIAM journal on scientific computing*, 39:A1851–A1878, 2017.

[24] M. Gharaei and A.J. Homburg. Random interval diffeomorphisms. *Discrete Contin. Dyn. Syst., Ser. S*, 10(2):241–272, 2017.

[25] P. Hafkenscheid. *Computing Braid Floer Homology*. PhD thesis, Vrije Universiteit Amsterdam, 2017.

[26] S. Hille, K. Horbacz, and T. Szarek. Existence of a unique invariant measure for a class of equicontinuous Markov operators with application to a stochastic model for an autoregulated gene. *Ann. Math. Blaise Pascal*, 23(2):171–217, 2016.

[27] A. Hoffman, H. Hupkes, and E. van Vleck. *Entire solutions for bistable lattice differential equations with obstacles*, volume 250. American Mathematical Society (Memoirs), 2017.

[28] A.J. Homburg. Synchronization in minimal iterated function systems on compact manifolds. *Bulletin of the Brazilian Mathematical Society, New Series*, Jan 2018.

[29] R. Jordan, D. Kinderlehrer, and F. Otto. The variational formulation of the Fokker–Planck Equation. *SIAM Journal on Mathematical Analysis*, 29(1):1–17, 1998.

[30] D.C. Koppenol, F.J. Vermolen, F.B. Niessen, P.P.M. van Zuijlen, and C. Vuik. A mathematical model for the simulation of the formation and the subsequent regression of hypertrophic scar tissue after dermal wounding. *Biomechanics and modeling in mechanobiology*, 16(1):15–32, February 2017.

[31] Q. Liu, A. Doelman, V. Rottschäfer, M. de Jager, P.M.J. Herman, M. Rietkerk, and J. van de Koppel. Phase separation explains a new class of self-organized spatial patterns in ecological systems. *Proceedings of the National Academy of Sciences*, 110(29):11905–11910, 2013.

[32] S. Verduyn Lunel. Using dynamics to analyse time series. In Pavel Gurevich, Juliette Hell, Björn Sandstede, and Arnd Scheel, editors, *Patterns of Dynamics*, pages 370–392, Cham, 2017. Springer International Publishing.

[33] A. Mielke, M.A. Peletier, and D.R.M. Renger. On the relation between gradient flows and the large-deviation principle, with applications to Markov chains and diffusion. *Potential Analysis*, 41(4):1293–1327, 2014.

[34] A. Mielke, M.A. Peletier, and D.R.M. Renger. A generalization of Onsager's reciprocity relations to gradient flows with nonlinear mobility. *Journal of Non-Equilibrium Thermodynamics*, 41(2):141–149, 2016.

[35] M. Muskulus and S. Verduyn Lunel. Wasserstein distances in the analysis of time series and dynamical systems. *Physica D*, 240(1):45–58, 2011.

[36] K.W. Myerscough, J. Frank, and B. Leimkuhler. Observation-based correction of dynamical models using thermostats. *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, 473(2197), 2017.

[37] K.W. Myerscough and J.E. Frank. Poisson integration of point vortices on the sphere. *Journal of Computational and Applied Mathematics*, 304:100–119, 2016.

[38] E. Nijholt, B. Rink, and J.A. Sanders. Graph fibrations and symmetries of network dynamics. *J. Differ. Equations*, 261:4861–4896, 2016.

[39] E. Nijholt, B. Rink, and J.A. Sanders. Center manifolds of coupled cell networks. *SIAM J. Math. Anal.*, 49(5):4117–4148, 2017.

[40] E. Nijholt, B. Rink, and J.A. Sanders. Projection blocks in homogeneous coupled cell networks. *Dynamical Systems*, 32(1):164–186, 2017.

[41] L. Onsager. Reciprocal relations in irreversible processes I & II. *Physical Review*, 37:405–426 and 38:2265–2279, 1931.

[42] E.G. Rens and R.H.M. Merks. Cell Contractility Facilitates Alignment of Cells and Tissues to Static Uniaxial Stretch. *Biophysical Journal*, 112(4):755–766, February 2017.

[43] B. Rink and J.A. Sanders. Amplified Hopf bifurcations in feed-forward networks. *SIAM J. Appl. Dyn. Syst.*, 12(2):1135–1157, 2013.

[44] B. Rink and J.A. Sanders. Coupled cell networks and their hidden symmetries. *SIAM J. Math. Anal.*, 46(2):1577–1609, 2014.

[45] B. Rink and J.A. Sanders. Coupled cell networks: semigroups, Lie algebras and normal forms. *Trans. Amer. Math. Soc.*, 367:3509–3548, 2015.

[46] W.H.A. Schilders and A. Lutowska. *Reduced Order Methods for Modeling and Computational Reduction*. Springer, Cham, 2014.

[47] M. Seslija, A.J. van der Schaft, and J.M.A. Scherpen. Discrete exterior geometry approach to structure-preserving discretization of distributed-parameter port-Hamiltonian systems. *Journal of Geometry and Physics*, 62(6):1509–1531, 2012.

[48] M. Seslija, A.J. van der Schaft, and J.M.A. Scherpen. Hamiltonian perspective on compartmental reaction–diffusion networks. *Automatica*, 50(3):737–746, 2014.

[49] L. Sewalt and A. Doelman. Spatially periodic multipulse patterns in a generalized Klausmeier–Gray–Scott model. *SIAM Journal on Applied Dynamical Systems*, 16(2):1113–1163, 2017.

[50] L. Sewalt, A. Doelman, H.G.E. Meijer, V. Rottschäfer, and A. Zagaris. Tracking pattern evolution through extended center manifold reduction and singular perturbations. *Physica D: Nonlinear Phenomena*, 298–299:48 – 67, 2015.

[51] A.H. Sheikh, D. Lahaye, L. Garcia Ramos, R. Nabben, and C. Vuik. Accelerating the shifted Laplace preconditioner for the Helmholtz equation by multilevel deflation. *Journal of Computational Physics*, 322:473–490, 2016.

[52] A.H. Sheikh, D. Lahaye, and C. Vuik. On the convergence of shifted Laplace preconditioner combined with multilevel deflation. *Numerical Linear Algebra with Applications*, 20:645–662, 2013.

[53] K. Siteur, E. Siero, M.B. Eppinga, J.D.M. Rademacher, A. Doelman, and M. Rietkerk. Beyond Turing: The response of patterned ecosystems to environmental change. *Ecological Complexity*, 20:81–96, 2014.

[54] K. Smetana and A.T. Patera. Optimal local approximation spaces for component-based static condensation procedures. *SIAM Journal on Scientific Computing*, 38(5):A3318–A3356, 2016.

[55] J.B. van den Berg, R. Ghrist, R.C. Vandervorst, and W. Wójcik. Braid Floer homology. *Journal of Differential Equations*, 259(5):1663–1721, 2015.

[56] A.J. van der Schaft and B.M. Maschke. Port-Hamiltonian systems on graphs. *SIAM Journal on Control and Optimization*, 51(2):906–937, 2013.

[57] A.J. van der Schaft, S. Rao, and B. Jayawardhana. On the mathematical structure of balanced chemical reaction networks governed by mass action kinetics. *SIAM Journal on Applied Mathematics*, 73(2):953–973, 2013.

[58] A.J. van der Schaft, S. Rao, and B. Jayawardhana. Complex and detailed balancing of chemical reaction networks revisited. *Journal of Mathematical Chemistry*, 53(6):1445–1458, 2015.

[59] S. van der Stelt, A. Doelman, G. Hek, and J.D.M. Rademacher. Rise and fall of periodic patterns for a generalized Klausmeier–Gray–Scott model. *Journal of Nonlinear Science*, 23(1):39–95, 2013.

[60] J.J.W. van der Vegt, F. Brink, and F. Iszak. Hamiltonian finite element discretization for nonlinear free surface water waves. *Journal of Scientific Computing*, 73:366–394, 2017.

[61] M.B. van Gijzen, Y.A. Erlangga, and C. Vuik. Spectral analysis of the discrete Helmholtz operator preconditioned with a shifted Laplacian. *SIAM Journal on Scientific Computing*, 29:1942–1958, 2007.

[62] S.A. van Gils, S. Janssens, Y. Kuznetsov, and S. Visser. On local bifurcations in neural field models with transmission delays. *Journal of mathematical biology*, 66(4–5):837–887, 2013.

[63] J.H. van Heerden, M.T. Wortel, F.J. Bruggeman, J.J. Heijnen, Y.J.M. Bollen, R. Planque, J. Hulshof, T.G. O'Toole, S.A. Wahl, and B. Teusink. Lost in Transition: Start-Up of Glycolysis Yields Subpopulations of Nongrowing Cells. *Science (New York, NY)*, 343(6174):1245114, February 2014.

[64] J. van Neerven, M. Veraar, and L. Weis. Stochastic maximal L^p-regularity. *Ann. Probab.*, 40(2):788–812, 2012.

[65] R.F.M. van Oers, E.G. Rens, D.J. LaValley, C.A. Reinhart-King, and R.H.M. Merks. Mechanical cell-matrix feedback explains pairwise and collective endothelial cell behavior in vitro. *PLoS computational biology*, 10(8):e1003774, 2014.

[66] H.J. van Waarde, M.K. Camlibel, and H.L. Trentelman. A distance-based approach to strong target control of dynamical networks. *IEEE Transactions on Automatic Control*, 62(12):6266–6277, 2017.

[67] D. Weihs, A. Gefen, and F.J. Vermolen. Review on experiment-based two and three-dimensional models for wound healing. *Interface Focus*, 6(5):20160038, August 2016.

[68] S. Zhuk, J.E. Frank, I. Herlin, and R. Shorten. Data assimilation for linear parabolic equations: minimax projection method. *SIAM Journal of Scientific Computing*, 37:A1174–A1196, 2015.

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Name of scientist	Expertise	HGL, UHD and UD	University
Christoph Brune	Variational Analysis, Medical Imaging	UD	UT
Bernard Geurts	Large Scale Computing	HGL	UT
Stephan van Gils	Delay equations, Neuroscience	HGL	UT/UU
Hil Meijer	Bifurcation theory, Neuroscience	UD	UT
Jaap van der Vegt	Numerical analysis	HGL	UT
Hans Zwart	Control of physical systems	HGL	UT
Bob Rink	Bifurcation theory, Network Dynamics	HGL	VU
Jan Bouwe van den Berg	Computer-assisted Theorems in Dynamics and PDEs	HGL	VU
Magnus Botnan	Topological Data Analysis	TT UD	VU
Bob Planqué	Mathematical Biology	UD	VU
Federica Pasquotto	Symplectic Topology	UD	VU
Oliver Fabert	Symplectic Geometry	UD	VU
Joost Hulshof	Applied Analysis	HGL	VU
Rob van der Vorst	Topological Dynamical Systems	HGL	VU
Ale Jan Homburg	Dynamical Systems	HGL	VU
Jan-Cees van der Meer	Multiscale & Transient Dynamics	UD	TU/e
Mark Peletier	Multiscale & Transient Dynamics	HGL	TU/e
Jim Portegies	Multiscale & Transient Dynamics	UD	TU/e
Georg Prokert	Multiscale & Transient Dynamics	UD	TU/e
Oliver Tse	Multiscale & Transient Dynamics	UD	TU/e
Martijn Anthonissen	Scientific Computing, PDEs	UD	TU/e
Björn Baumeier	Scientific Computing, Molecular Dynamics, Quantum Chemistry	UD	TU/e
Michiel Hochstenbach	Scientific Computing, Numerical Linear Algebra, Big Data	UHD	TU/e
Laura Iapichino	Scientific Computing, Model Reduction	UD	TU/e
Barry Koren	Scientific Computing, CFD	HGL	TU/e
Jos Maubach	Scientific Computing, PDEs	UD	TU/e
Wil Schilders	Scientific Computing, Model Reduction	HGL	TU/e
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Andrea Fuster	Mathematical Imaging	UD	TU/e
Jan Brandts	Numerical Analysis, Numerical Linear Algebra	UHD	UvA
Ale Jan Homburg	Dynamical Systems	UHD	UvA
Han Peters	Complex Dynamics	UHD	UvA
Rob Stevenson	Numerical Analysis	HGL	UvA
Chris Stolk	Numerical Analysis, Inverse Problems	UHD	UvA
Jan Wiegerinck	Complex Analysis	HGL	UvA
Holger Waalkens	Hamiltonian systems and semiclassical quantum mechanics	HGL	RUG
Christobal Bertoglio	Computational methods for medical applications/imaging	TT UD	RUG
Konstantinos Efstathiou	Geometry of integrable Hamiltonian fibrations, chaos and diffusion, networks	TT UD	RUG
Alef Sterk	Bifurcation theory, applications to climate models, analysis	TT UD	RUG
Roel Verstappen	Computational Fluid Dynamics, multi-scale simulations	HGL	RUG
Fred Wubs	Numerical bifurcation analysis of PDEs	UHD	RUG
Kanat Camlibel	Systems and control	HGL	RUG
Arjan van der Schaft	Systems and control	HGL	RUG
Harry Trentelman	Systems and control	HGL	RUG
Alden Waters	Partial Differential Equations and applications	TT UD	RUG

Daan Crommelin	Stochastic multiscale modeling, Uncertainty quantification	HGL	CWI
Ute Ebert	Multiscale modeling, Numerical methods for plasma physics	HGL	CWI
Kees Oosterlee	Computational finance, Numerical analysis	HGL	CWI
Joost Batenburg	Computational imaging and tomography	HGL	CWI
Enrico Camporeale	Numerical methods for plasma physics, Machine Learning	UD	CWI
Benjamin Sanderse	Uncertainty quantification, Computational fluid dynamics	TT	CWI
Svetlana Dubinkina	Data assimilation, Computational fluid dynamics	TT	CWI
Felix Lucka	Mathematics and Algorithms for 3D Imaging of Dynamic Processes	TT	CWI
Arnold Heemink	Stochastic Differential Equations	HGL	TUD
Wim van Horssen	Partial Differential Equations	UHD	TUD
Hai Xiang Lin	Scientific Computing, Parallel Computing	UHD	TUD
Henk Schuttelaars	Shallow Water Equations	UD	TUD
Martin Verlaan	Data Assimilation, Shallow Water Equations	HGL	TUD
Johan Dubbeldam	Dynamical Systems	UD	TUD
Jacob Van der Woude	Control Theory	UHD	TUD
Bernard Meulenbroek	Porous Media Flow	UD	TUD
Kees Vuik	Scientific Computing, Partial Differential Equations	HGL	TUD
Kees Oosterlee	Scientific Computing, Partial Differential Equations	HGL	TUD
Martin van Gijzen	Scientific Computing, Image Processing	UHD	TUD
Fred Vermole	Health, Partial Differential Equations	UHD	TUD
Domenico Lahaye	Scientific Computing, Industrial Problems	UD	TUD
Jennifer Ryan	Discontinuous Galerkin	UD	TUD
Duncan van der Heul	Computational Fluid Dynamics	UD	TUD
Neil Budko	Scientific Computing, Industrial Problems	UD	TUD
Eric Deleersnijder	Computational Fluid Dynamics	HGL	TUD
emus Hanea	Data Assimilation, Shallow Water Equations	UD (onbezoldigd)	TUD
Ramses van der Toorn	Partial Differential Equations	UD	TUD
Arjen Doelman	partial differential equations, dynamical systems, stability theory, applications in ecology	HGL	LU
Roeland Merks	scientific computing, multiple scales, stochastic dynamics	HGL	LU
Marcel de Jeu	abstract harmonic analysis, representation theory, ordered vector spaces and algebras, special functions	UHD	LU
Vivi Rottschäfer	partial differential equations, dynamical systems, geometric singular perturbation theory, applications from pharmacology and ecology	UHD	LU
Hermen Jan Hupkes	lattice differential equations, delay differential equations, discretization schemes, noisy patterns, economic modeling	UHD	LU
Martina Chirilus-Bruckner	nonlinear partial differential equations, waves and patterns, dynamical systems, modulation equations, inverse spectral theory	TT UD	LU
Onno van Gaans	stochastic differential equations, partially ordered vector spaces, delay differential equations	UD	LU
Sander Hille	dynamical systems, stochastic perturbation, mathematical biology	UD	LU
Rob Bisseling	high-performance computing, sparse matrix computations, bioinformatics, combinatorial methods	HGL	UU
Jason Frank	numerical analysis, multiscale modelling, data assimilation, complex systems	HGL	UU

Heinz Hanßmann	Hamiltonian mechanics, integrable systems	UD	UU
Carolin Kreisbeck	variational methods, homogenization, multiscale modelling of materials	UD	UU
Yuri Kuznetsov	bifurcation theory and computation	HGL	UU/UT
Tristan van Leeuwen	numerical analysis of inverse problems, seismic inversion, medical imaging	UD	UU
Sjoerd Verduyn Lunel	functional and delay differential equations, applied analysis, time-series analysis, mathematical biology	HGL	UU
Paul Zegeling	numerical methods for PDEs, moving mesh methods, numerical methods for fractional DEs, flows in porous media	UHD	UU
Stefanie Sonner	infinite dimensional dynamical systems	UD	RUN