Analysis and Numerics of Nonlinear PDEs: Degeneracies & Free Boundaries
July 3—7 2023 Lorentz Center@Snellius

Scientific Description and Aims
The aim of the workshop was to bring together scientists from the Netherlands and abroad with expertise in three main areas: problems of porous-medium type, thin-film equations, and free-boundary problems. We aimed at both advancing the current state of the art and forming a network of experts in this field. Moreover, we wanted to foster new collaborations between mathematicians from different mathematical communities and to connect the theoretical analysis with numerical mathematics and real-world applications.

Tangible Outcomes.
The workshop served as a great opportunity for several young researchers from the Netherlands and abroad to present their research to an internationally distinguished audience and to enter a network of experts in their field. It was also a starting point for new collaborations. We identified multiple open problems in the three main research areas of the participants and on the interface between them. Due to both the composition of the participants with different expertise and a schedule with a lot of time for interactions, we were able to intensively discuss open problems from different viewpoints. This allowed in many cases for a rigorous mathematical formulation of these problems and even for identifying possible approaches to tackle them. Examples:

- existence and non-existence of traveling wave solutions for porous-medium type equations (for different parameter regimes)
- numerical and analytical investigation of possible instabilities of thin-film flows down a fiber
- well-posedness of the thin-film free boundary problem for different mobility exponents in different wetting regimes
- a non-existence proof for the thin-film free-boundary problem with cubic mobility (corresponding to no slip at the liquid-solid interface)
- de-wetting of solutions to a thin-film equation in finite time
- well-posedness and qualitative behavior of fourth-order biofilm models

New scientific contacts and collaborations between the workshop participants were initiated within groups of researchers of the same community, as well as within groups from different areas. Since both participants and organizers were enthusiastic about the outcomes of the week, follow-up workshops are envisaged in the Netherlands or abroad.

Scientific Breakthroughs
There are some challenging problems in the field of the workshop that are open for more than 30 years: they were intensively discussed within the problem sessions. A precise mathematical formulation was obtained for parts of these problems and approaches to solve them were identified. The workshop participants want to stay in contact and share further progress.
We envisaged connections between approaches for systems of porous-medium type modelling biofilms and thin-film equations. Transfer of knowledge was achieved in discussions among participants from both communities. Relevant literature was found and discussed providing the basis for new collaborations.

“Aha” Moments
The atmosphere was extremely friendly and “gezellig”. To mention one episode, on Wednesday morning a heavy storm hit the Netherlands. Now, of course there is nothing to laugh about it, but we must admit that hanging dripping dresses and shoes all around the conference area and lecturing with naked feet brought us some funny times.

Organization
Physical workshop with additional online participants during talks. The discussions took place only among physical participants, while online participants could follow all talks.

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